EVOLUTION OF ANNULAR LIGHT BEAMS IN A NONLINEAR MEDIUM

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The question of the behavior of a stationary annular light beam in a nonlinear medium for the case $\partial n/\partial E < 0$ was first investigated theoretically and experimentally by G. A. Askar'yan and V. B. Studenov [1]. A numerical investigation is also carried out herein for the case $\partial n/\partial E > 0$.

As is known (see [2, 3], for example), the problem reduces to finding the field amplitude in specific approximations from the equation

$$2i\frac{\partial u}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + [f_1(|u|^2) - if_2(|u|^2)]\,u \tag{1}$$

with the boundary conditions

$$\frac{\partial u}{\partial r}(0, z) = 0, \quad u\ (\infty, z) = 0 \tag{2}$$

Here f_1 and f_2 characterize the nonlinear and dissipative properties of the medium. Annular beams of the form

$$u(r, 0) = a_0 r^{\alpha} \exp[-(r - r_0)^2 / l^2] \tag{3}$$

were selected as the initial distribution.

Equation (1) cannot be solved analytically, which obliges reliance on numerical methods. The method of computation described in [4, 5] is used. Several kinds of media were examined.

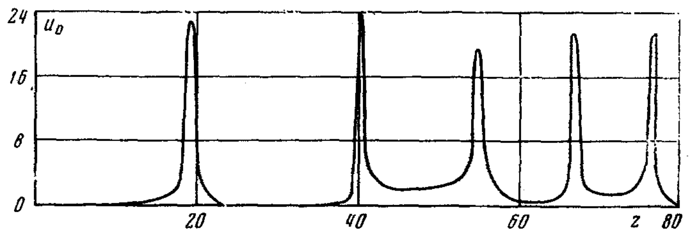

Fig. 1

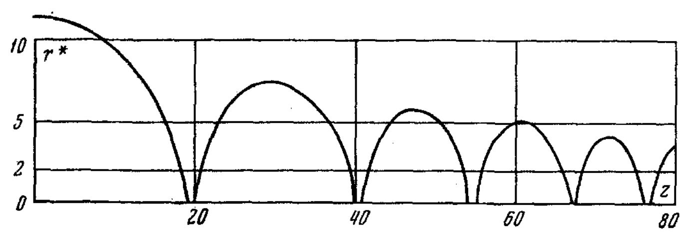

Fig. 2

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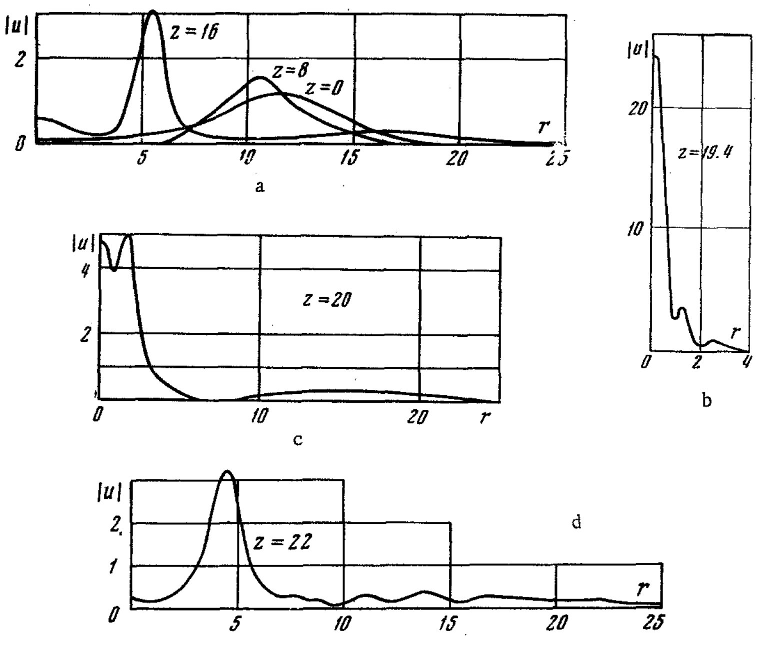

Fig. 3

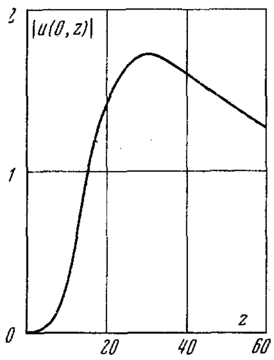

Fig. 4

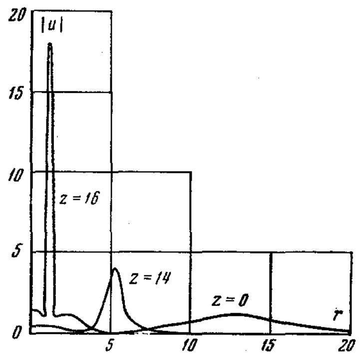

Fig. 5

1. Medium with Nonlinearity Saturation. Here we selected

$$f_1 = \frac{\sigma}{\varkappa}\left(1 - \exp\left(-\varkappa |u|^2\right)\right), \qquad f_2 = 0$$

All the figures presented henceforth refer to the case $\sigma = 1$, $\varkappa = 0.1$. The results can be described qualitatively as follows.

The annulus performs radial oscillations, where the annular structure even vanishes at certain times: the maximum $|u|$ lies on the axis and $|u|$ drops almost monotonely as r grows (a centered beam). A tendency to stabilization is hence noticeable: the amplitude fluctuations on the axis and the span of the radial oscillations diminish. Presented in Figs. 1, 2, and 3 are results for the case $a_0 = 0.01$, $\alpha = 2$, $l = 4$, $r_0 = 10$. Here the power P of the light beam exceeds the critical value P_* by 40 times [2, 3]. Shown in Fig. 1 is the z-dependence of the field amplitude $u_0 \equiv |u(0, z)|$ on the axis. The variable r^* in Fig. 2 denotes the radial coordinate of the point where the field $|u|$ is maximal. Several typical $|u|$ profiles are pictured in Fig. 3 for different z, where the "collapse" of the ring in phase is shown in Fig. 3a, b, and its expansion in phase is shown in Fig. 3c, d. The behavior of the field on the axis in the case of pure diffraction ($f_1 = f_2 = 0$) is presented for comparison in Fig. 4.

The results presented favor the hypothesis, expressed in [5], about the light beam stabilization in a noncubic conservative medium with negative invariant

$$J = \int_0^\infty \{|\nabla u|^2 - F(|u|^2)\}\, r\, dr, \qquad F(\xi) = \int_0^\xi f_1(\eta)\, d\eta$$

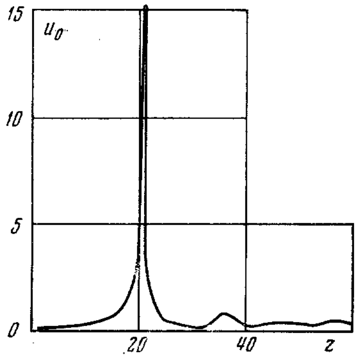

Fig. 6

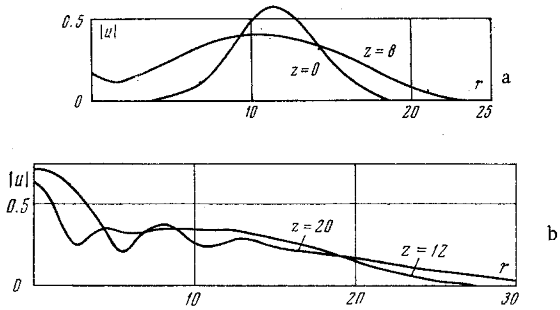

Fig. 7

The possibility of the formation of an annular structure from a centered beam is curious. Naturally, the essential part of the information on the future of the beam is included in its phase distribution. An annulus from an initially centered beam of special form was obtained successfully even for a constant initial phase.

2. A Purely Cubical Medium ($f_1 = \sigma|u|^2, f_2 = 0$). Here the annular light beam will collapse for $P > P_*$. The maximum field amplitude hence grows monotonely, and the leading front (directed towards the beam axis) becomes circular. The picture of an annular collapsing beam is presented in Fig. 5.

The influence of two-photon absorption on the propagation of an annular beam in a cubical medium ($f_1 = \sigma|u|^2, f_2 = \chi|u|)^2$ was also investigated. As should have been expected, absorption hinders beam collapse. Damped axial and radial oscillations are observed instead. As χ grows, the frequency of the radial oscillations diminishes. In the limit case of large absorption, only one maximum is observed on the axis (Fig. 6, $\chi = 0.01$).

As regards the case $\partial n/\partial E < 0$, the numerical experiments verify the results in [1] (see Fig. 7a, b; here $a_0 = 0.005$, $\alpha = 2$, $l = 4$, $r_0 = 10$, $P \approx 10\ P_*$).

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